Algèbre, page 23, exercices 1-2, équations du deuxième degré à une inconnue, méthode du discriminant Δ, corrigé détaillé :

1. (a) $x^2 + x - 1 = 0$
$a = 1,\ b = 1,\ c = -1$
$\Delta = 1^2 - 4 \cdot 1 \cdot (-1) = 5 > 0$
$x = \frac{-1-\sqrt{5}}{2}$ ou $x = \frac{-1+\sqrt{5}}{2}$
$S = \left\{\frac{-1-\sqrt{5}}{2}; \frac{-1+\sqrt{5}}{2}\right\}$

(b) $4x^2 - 20x = -25$
$4x^2 - 20x + 25 = 0$
$a = 4,\ b = -20,\ c = 25$
$\Delta = 20^2 - 4 \cdot 4 \cdot 25 = 0$
$x = \frac{-(-20)}{2 \cdot 4} = \frac{5}{2}$
$S = \left\{\frac{5}{2}\right\}$

(c) $-6x + x^2 + 4 = 0$
$x^2 - 6x + 4 = 0$
$a = 1,\ b = -6,\ c = 4$
$\Delta = (-6)^2 - 4 \cdot 1 \cdot 4 = 20 > 0$
$x = \frac{-(-6)-\sqrt{20}}{2}$ ou $x = \frac{-(-6)+\sqrt{20}}{2}$
$x = \frac{6-2\sqrt{5}}{2}$ ou $x = \frac{6+2\sqrt{5}}{2}$
$x = \frac{2(3-\sqrt{5})}{2}$ ou $x = \frac{2(3+\sqrt{5})}{2}$
$x = 3 - \sqrt{5}$ ou $x = 3 + \sqrt{5}$
$S = \left\{3 - \sqrt{5}; 3 + \sqrt{5}\right\}$

(d) $3x^2 - x = 30$
$3x^2 - x - 30 = 0$
$a = 3,\ b = -1,\ c = -30$
$\Delta = (-1)^2 - 4 \cdot 3 \cdot (-30) = 361 > 0$
$x = \frac{-(-1)-\sqrt{361}}{2 \cdot 3}$ ou $x = \frac{-(-1)+\sqrt{361}}{2 \cdot 3}$
$x = \frac{1-19}{6}$ ou $x = \frac{1+19}{6}$
$x = -3$ ou $x = \frac{10}{3}$
$S = \left\{-3; \frac{10}{3}\right\}$

(e) $-2x^2 - 10x + 61 = 0$
$a = -2,\ b = -10,\ c = 61$
$\Delta = (-10)^2 - 4 \cdot (-2) \cdot 61 = 588 > 0$
$x = \frac{-(-10)-\sqrt{588}}{2 \cdot (-2)}$ ou $x = \frac{-(-10)+\sqrt{588}}{2 \cdot (-2)}$
$x = \frac{10-14\sqrt{3}}{-4}$ ou $x = \frac{10+14\sqrt{3}}{-4}$
$x = \frac{2(5-7\sqrt{3})}{-4}$ ou $x = \frac{2(5+7\sqrt{3})}{-4}$
$x = \frac{-5+7\sqrt{3}}{2}$ ou $x = \frac{-5-7\sqrt{3}}{2}$
$S = \left\{\frac{-5-7\sqrt{3}}{2}; \frac{-5+7\sqrt{3}}{2}\right\}$

(f) $9x^2 - 30x + 25 = 0$
$a = 9,\ b = -30,\ c = 25$
$\Delta = (-30)^2 - 4 \cdot 9 \cdot 25 = 0$
$x = \frac{-(-30)}{2 \cdot 9} = \frac{5}{3}$
$S = \left\{\frac{5}{3}\right\}$

(g) $(x-6)(x+1) - (2x+3)(x-5) = 0$
$x^2 - 5x - 6 - \left(2x^2 - 7x - 15\right) = 0$
$-x^2 + 2x + 9 = 0$
$a = -1,\ b = 2,\ c = 9$
$\Delta = 2^2 - 4 \cdot (-1) \cdot 9 = 40 > 0$
$x = \frac{-2-\sqrt{40}}{2 \cdot (-1)}$ ou $x = \frac{-2+\sqrt{40}}{2 \cdot (-1)}$
$x = \frac{-2-2\sqrt{10}}{-2}$ ou $x = \frac{-2+2\sqrt{10}}{-2}$
$x = \frac{-2(1+\sqrt{10})}{-2}$ ou $x = \frac{-2(1-\sqrt{10})}{-2}$
$x = 1 + \sqrt{10}$ ou $x = 1 - \sqrt{10}$
$S = \left\{1 - \sqrt{10}; 1 + \sqrt{10}\right\}$

(h) $x - 7 = 6 - (x-7)^2$
$(x-7)^2 + x - 7 - 6 = 0$
$x^2 - 14x + 49 + x - 13 = 0$
$x^2 - 13x + 36 = 0$
$a = 1,\ b = -13,\ c = 36$
$\Delta = (-13)^2 - 4 \cdot 1 \cdot 36 = 25 > 0$
$x = \frac{-(-13)-\sqrt{25}}{2}$ ou $x = \frac{-(-13)+\sqrt{25}}{2}$
$x = \frac{13-5}{2}$ ou $x = \frac{13+5}{2}$
$x = 4$ ou $x = 9$
$S = \{4; 9\}$

(i) $(x-5)^3 - (x+2)^3 + 91 = 0$
$x^3 - 15x^2 + 75x - 125 - \left(x^3 + 6x^2 + 12x + 8\right) + 91 = 0$
$x^3 - 15x^2 + 75x - 125 - x^3 - 6x^2 - 12x - 8 + 91 = 0$
$-21x^2 + 63x - 42 = 0$
$x^2 - 3x + 2 = 0$
$a = 1,\ b = -3,\ c = 2$
$\Delta = (-3)^2 - 4 \cdot 1 \cdot 2 = 1 > 0$
$x = \frac{-(-3)-\sqrt{1}}{2}$ ou $x = \frac{-(-3)+\sqrt{1}}{2}$
$x = \frac{3-1}{2}$ ou $x = \frac{3+1}{2}$
$x = 1$ ou $x = 2$
$S = \{1; 2\}$

(j) $\frac{3x-7}{5} + \frac{x^2-9}{7} = 2$
$7(3x-7) + 5\left(x^2 - 9\right) = 70$
$21x - 49 + 5x^2 - 45 = 70$
$5x^2 + 21x - 164 = 0$
$a = 5,\ b = 21,\ c = -164$
$\Delta = (21)^2 - 4 \cdot 5 \cdot (-164) = 3721 > 0$
$x = \frac{-21-\sqrt{3721}}{2 \cdot 5}$ ou $x = \frac{-21+\sqrt{3721}}{2 \cdot 5}$
$x = \frac{-21-61}{10}$ ou $x = \frac{-21+61}{10}$
$x = -\frac{41}{5}$ ou $x = 4$
$S = \left\{-\frac{41}{5}; 4\right\}$

2. (a) $91x^2 = 2x + 45$
$91x^2 - 2x - 45 = 0$
$a = 91,\ b = -2,\ c = -45$
$\Delta = (-2)^2 - 4 \cdot 91 \cdot (-45) = 16384 > 0$
$x = \frac{-(-2)-\sqrt{16384}}{2 \cdot 91}$ ou $x = \frac{-(-2)+\sqrt{16384}}{2 \cdot 91}$
$x = \frac{2-128}{2 \cdot 91}$ ou $x = \frac{2+128}{2 \cdot 91}$
$x = -\frac{9}{13}$ ou $x = \frac{5}{7}$
$S = \left\{-\frac{9}{13}; \frac{5}{7}\right\}$

(b) $7x + 2x^2 = -7$
$2x^2 + 7x + 7 = 0$
$a = 2,\ b = 7,\ c = 7$
$\Delta = 7^2 - 4 \cdot 2 \cdot 7 = -7 < 0$
$S = \emptyset$

(c) $20x^2 + 117x - 161 = 0$
$a = 20,\ b = 117,\ c = -161$
$\Delta = (117)^2 - 4 \cdot 20 \cdot (-161) = 26569 > 0$
$x = \frac{-117-\sqrt{26569}}{2 \cdot 20}$ ou $x = \frac{-117+\sqrt{26569}}{2}$
$x = \frac{-117-163}{2 \cdot 20}$ ou $x = \frac{-117+163}{2 \cdot 20}$
$x = -7$ ou $x = \frac{23}{20}$
$S = \left\{-7; \frac{23}{20}\right\}$

(d) $4(x-3) + x(x-5) - 30 = 0$
$4x - 12 + x^2 - 5x - 30 = 0$
$x^2 - x - 42 = 0$
$a = 1,\ b = -1,\ c = -42$
$\Delta = (-1)^2 - 4 \cdot 1 \cdot (-42) = 169 > 0$
$x = \frac{-(-1)-\sqrt{169}}{2}$ ou $x = \frac{-(-1)+\sqrt{169}}{2}$
$x = \frac{1-13}{2}$ ou $x = \frac{1+13}{2}$
$x = -6$ ou $x = 7$
$S = \{-6; 7\}$

(e) $(5x-1)^2 + x^2 + 3 = 0$
$25x^2 - 10x + 1 + x^2 + 3 = 0$
$26x^2 - 10x + 4 = 0$
$a = 26,\ b = -10,\ c = 4$
$\Delta = (-10)^2 - 4 \cdot 26 \cdot 4 =< 0$
$S = \emptyset$

(f) $(x+1)^2 - (x-1)^2 = (x-8)^2$
$x^2 + 2x + 1 - \left(x^2 - 2x + 1\right) = x^2 - 16x + 64$
$x^2 + 2x + 1 - x^2 + 2x - 1 - x^2 + 16x - 64 = 0$
$-x^2 + 20x - 64 = 0$
$a = -1,\ b = 20,\ c = -64$
$\Delta = 20^2 - 4 \cdot (-1) \cdot (-64) = 144 > 0$
$x = \frac{-20-\sqrt{144}}{2 \cdot (-1)}$ ou $x = \frac{-20+\sqrt{144}}{2 \cdot (-1)}$
$x = \frac{-20-12}{-2}$ ou $x = \frac{-20+12}{-2}$
$x = 16$ ou $x = 4$
$S = \{4; 16\}$

(g) $\frac{x^2-3}{2} - \frac{x^2+1}{3} = \frac{x^2-11}{6}$
$3\left(x^2 - 3\right) - 2\left(x^2 + 1\right) = x^2 - 11$
$3x^2 - 9 - 2x^2 - 2 - x^2 + 11 = 0$
$0 = 0$
$S = \mathbb{R}$

(h) $\frac{(x-2)^2}{5} = \frac{(x-3)^2}{4}$
$4(x-2)^2 = 5(x-3)^2$
$4\left(x^2 - 4x + 4\right) - 5\left(x^2 - 6x + 9\right) = 0$
$4x^2 - 16x + 16 - 5x^2 + 30x - 45 = 0$
$-x^2 + 14x - 29 = 0$
$a = -1,\ b = 14,\ c = -29$
$\Delta = 14^2 - 4 \cdot (-1) \cdot (-29) = 80 > 0$
$x = \frac{-14-\sqrt{80}}{2 \cdot (-1)}$ ou $x = \frac{-14+\sqrt{80}}{2 \cdot (-1)}$
$x = \frac{-14-4\sqrt{5}}{-2}$ ou $x = \frac{-14+4\sqrt{5}}{-2}$
$x = \frac{-2(7+2\sqrt{5})}{-2}$ ou $x = \frac{-2(7-2\sqrt{5})}{-2}$
$x = 7 + 2\sqrt{5}$ ou $x = 7 - 2\sqrt{5}$
$S = \left\{7 - 2\sqrt{5}; 7 + 2\sqrt{5}\right\}$